Civil Engineering

(Highlighting the concepts of Structural Analysis)

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Contents

Chapter-1: *Emphasis upon the importance of Analysis & Theory of Structures*

Civil Engineering is a branch of engineering which deals with analysis, design, construction and maintenance of buildings, roads, bridges, dams and more of similar structures including the sub-structures of main structural elements. It is also necessary to study the loading standards to be adopted for the known dimensional geometry of the structure. In a routine procedure, Architect provides the shape, dimensions and geometry of the structure to be designed and finally constructed.

As far as learning for a degree course is concerned, a Civil Engineer has to study several subjects. These are Building design and construction, Railway Engineering, Highway Engineering (Including pavement design, run-ways, flyovers), Irrigation Engineering and public Health Engineering. In most of the Universities, emphasis is laid upon Structural Engineering. Design of R.C.C. , Prestressed Concrete and Steel Structures.

All these subjects require the knowledge of Theory of structures. In this subject, usually 2 things are given. These are loadings and the form and the Geometry of the structure. Students are required to find out maximum Thrust (Compressive forces), maximum Pull (Tensile forces), maximum Shearing force and maximum Bending moment for the member of a given frame.

1.1. "Mechanics" or "Theory" of Structures

The Subject of Engineering Mechanics is taught at First Year level. And the students are familiar with drawing of Shearing Force and Bending Moment diagrams. They are also familiar with the sign conventions i.e. which is positive and which is negative. Students often get confused when they draw Bending Moment Diagrams about the positive and negative side of the diagrams. Since majority of the buildings are made of R.C.C., the main reinforcement is provided on the tension side.

Therefore it becomes necessary to draw the Bending Moment Diagrams on Tension side so that the main steel is placed easily. If the diagram is on the topside, the main reinforcement will be put on the top side.

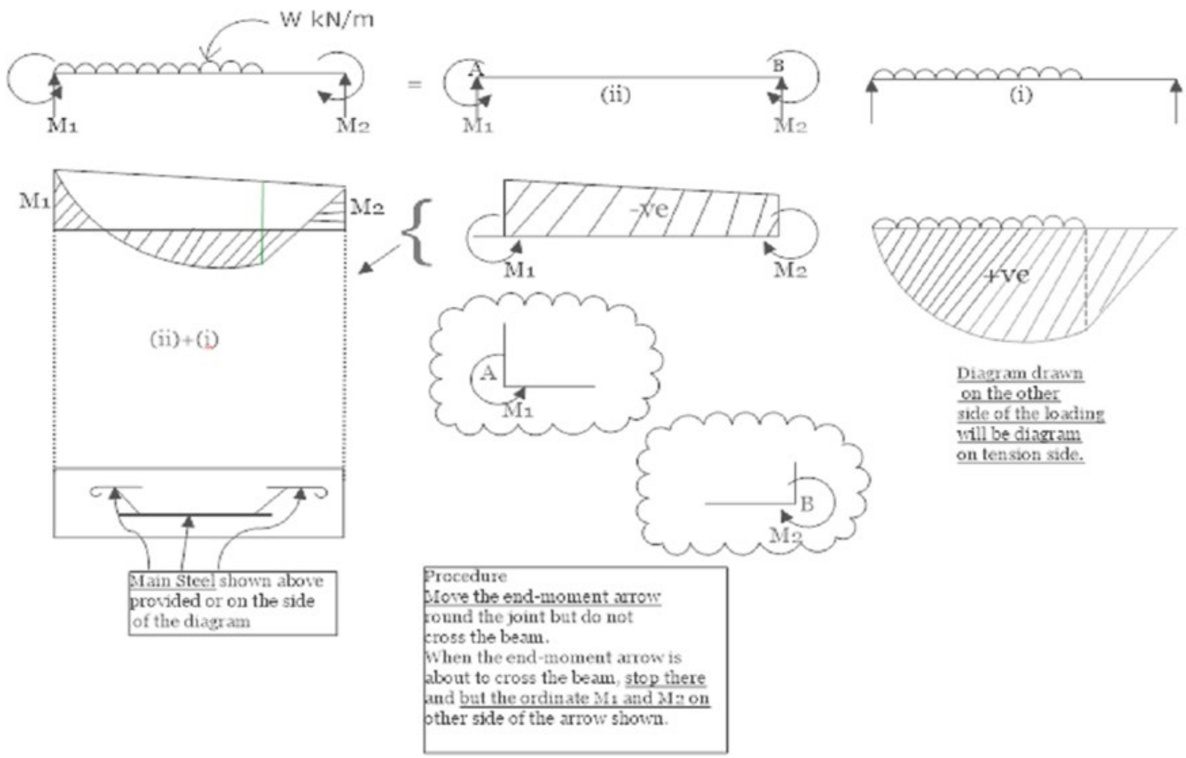
1.2. How to draw the diagram on Tension Side

Every member has mainly two types of loadings.

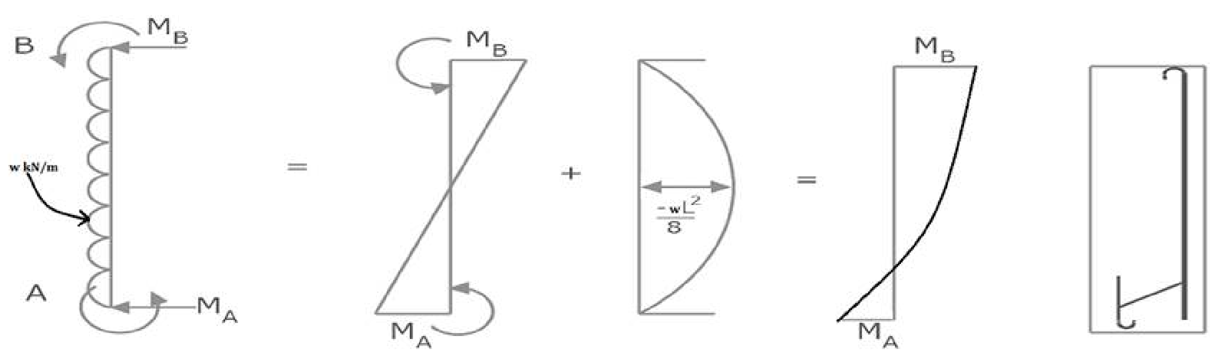
(i) Loading on the members itself

(ii) End Moments on the member calculated by theory of structures

On a beam shown below, both of the above loadings are shown. To make it understand easily, we will assume a portion of the uniformly distributed load. Both of these loadings are shown together and also separately.



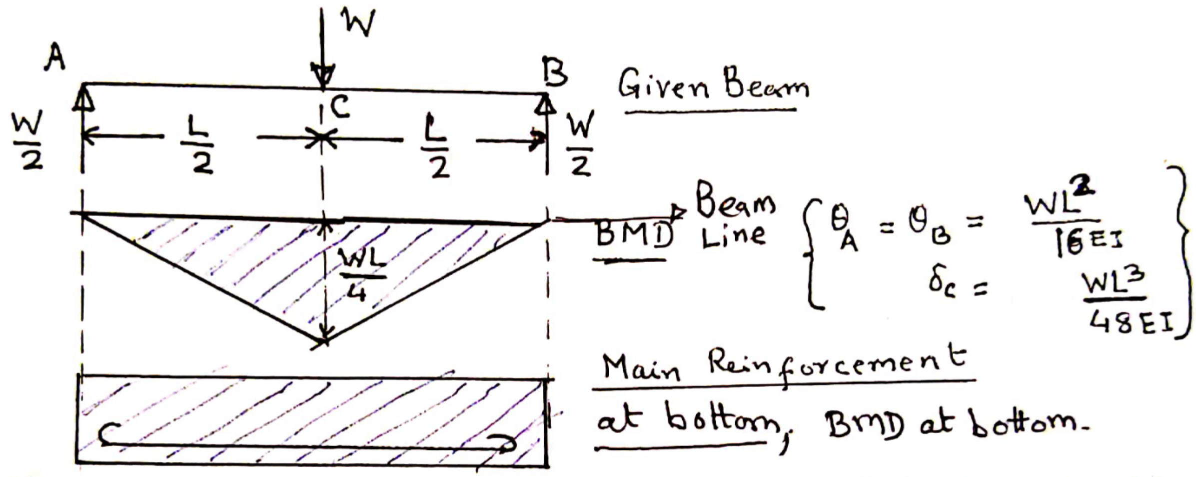
A vertical column-AB



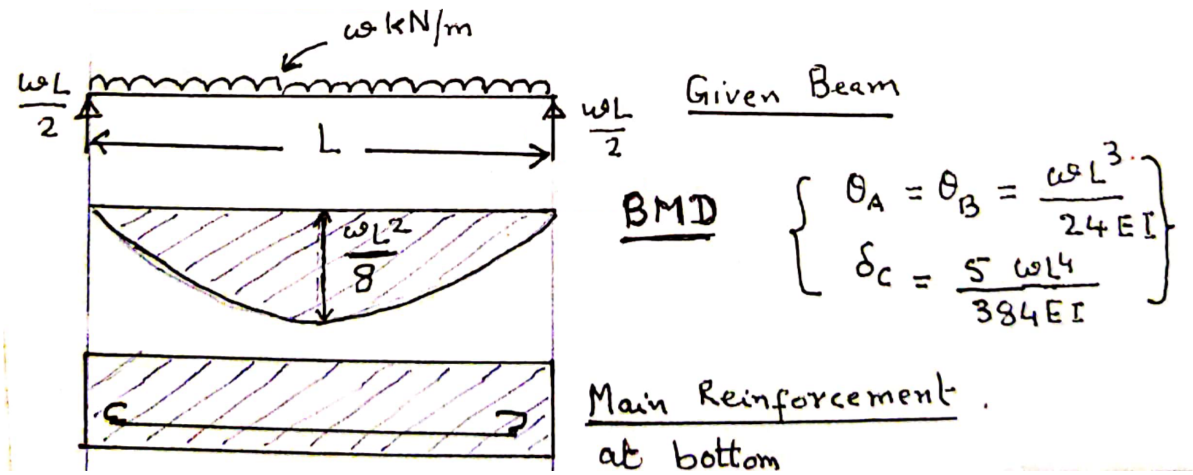
Note: Reactions are not shown in above diagrams.

1.3. Standard Loading Problems with important parameters

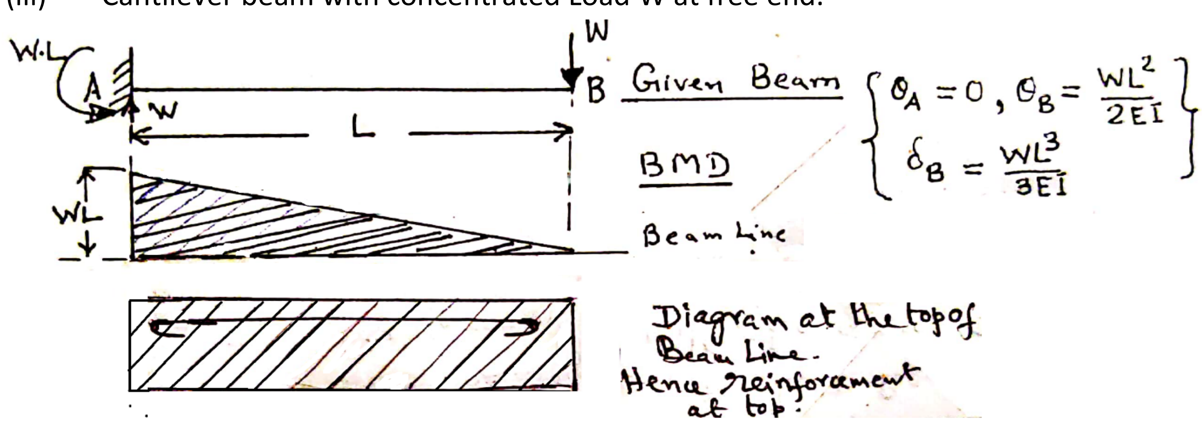
(i) Simple beam with central load W, Span = L



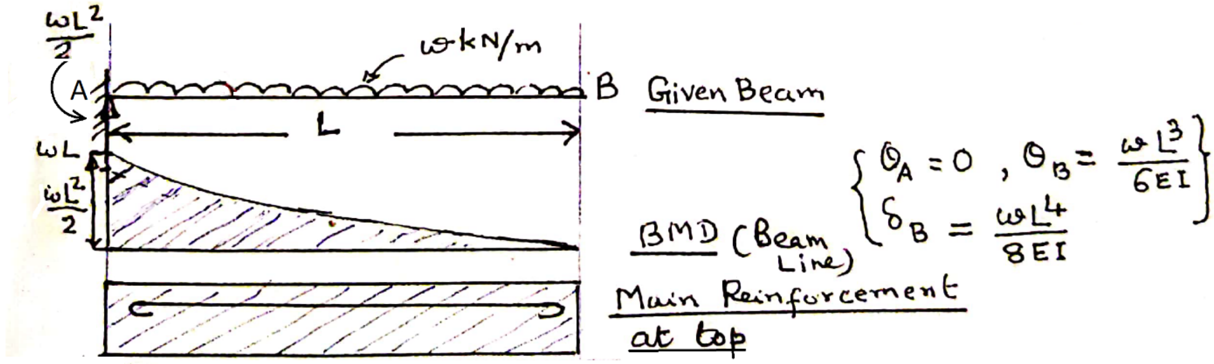
(ii) Simple beam with uniformly distributed load of wkN/m



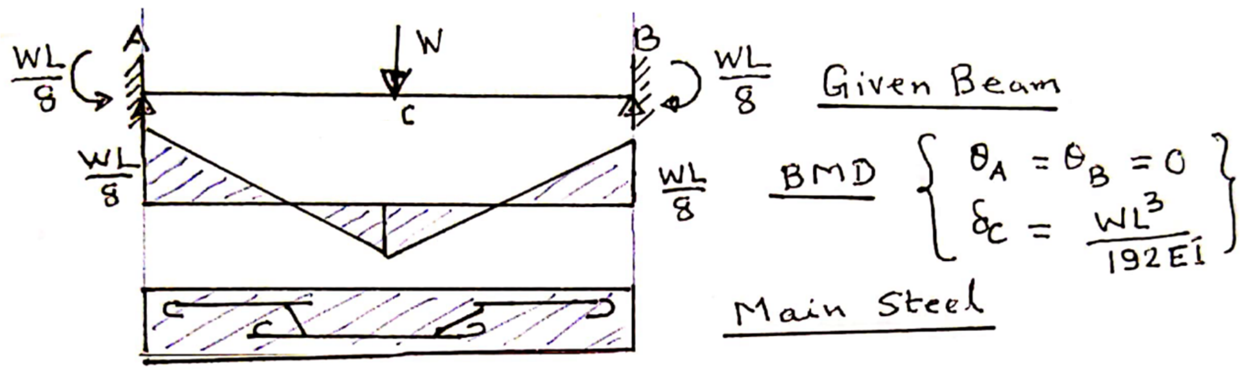
(iii) Cantilever beam with concentrated Load W at free end.



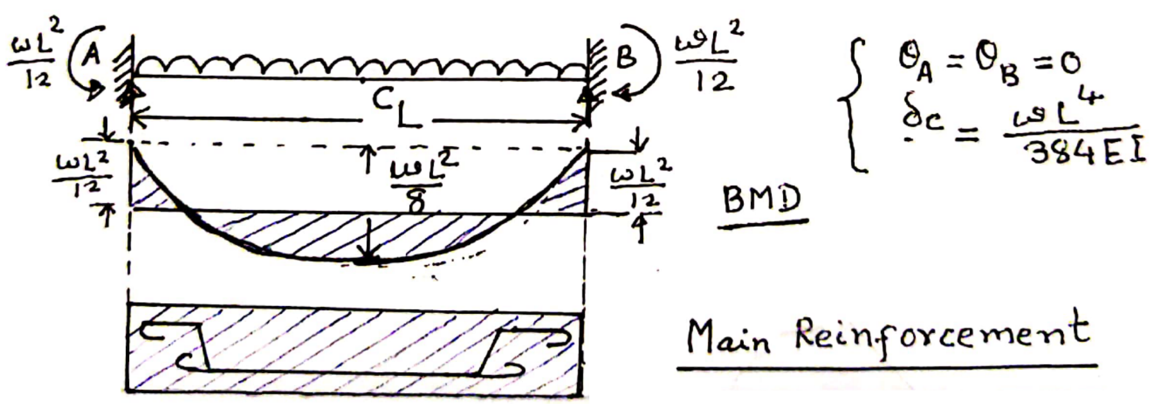
(iv) Cantilever beam with uniformly distributed load wkN/m



(v) Fixed beam with load W at center of Span = L

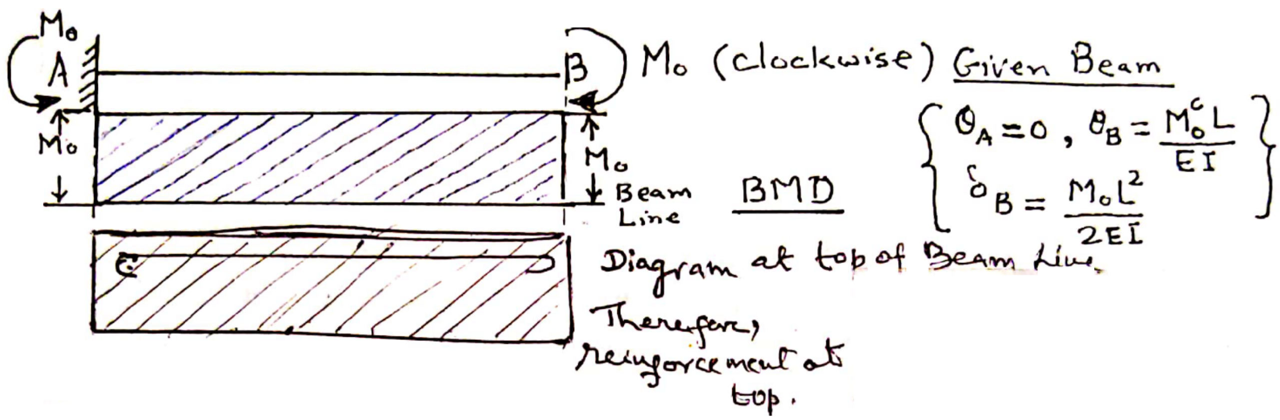


(vi) Fixed beam with uniformly distributed load wkN/m on full span

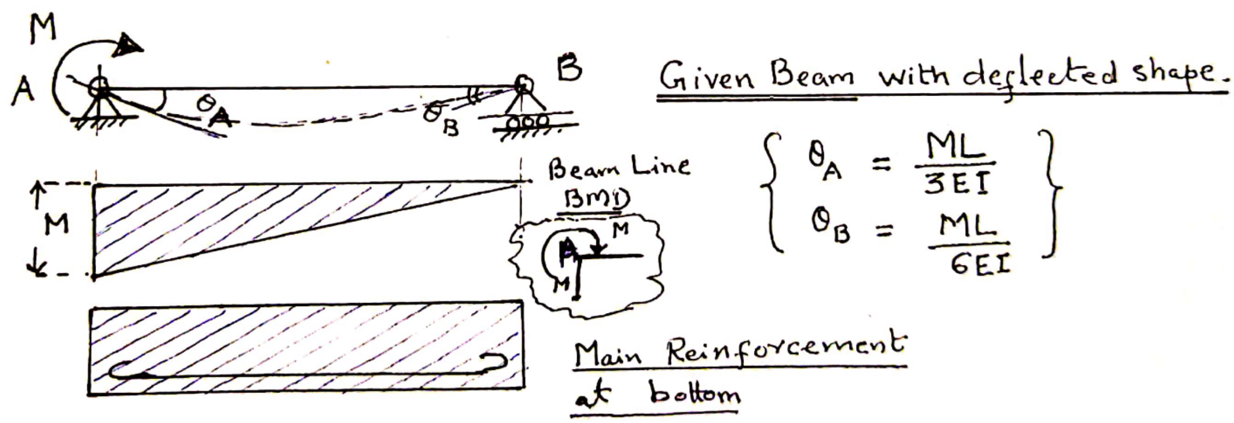


1.4. Moment Loading - Problems

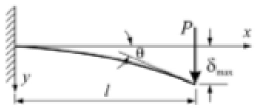
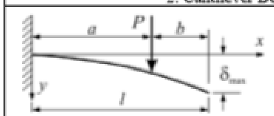
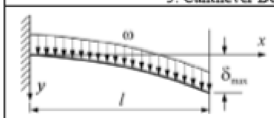
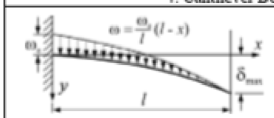
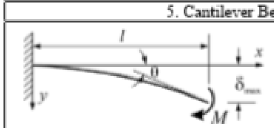
(vii) Cantilever beam with moment Mo at free end



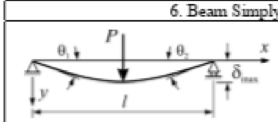
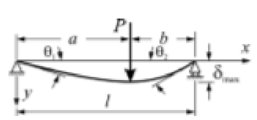
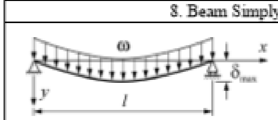
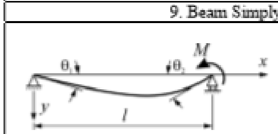
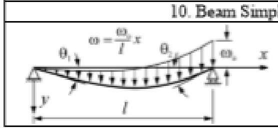
(viii) A simply supported beam with a clockwise moment "M" at one end



BEAM DEFLECTION FORMULAE

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

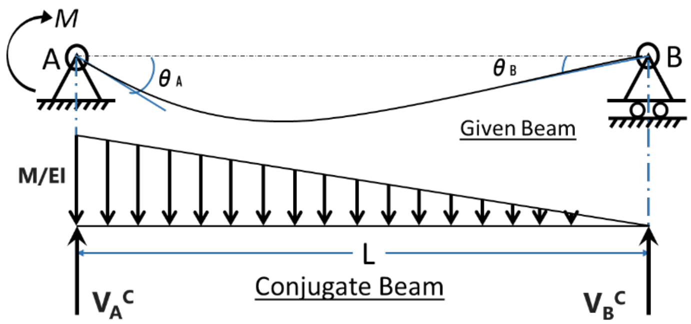
BEAM DEFLECTION FORMULAS

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}(l^2 - x^2 - b^2)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x - a)^3 + (l^2 - b^2)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{(l^2 - b^2)/3}$ $\delta = \frac{Pb}{48EI}(3l^2 - 4b^2)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \frac{\omega l^3}{24EI}$	$y = \frac{\omega x}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \frac{Ml}{6EI}$ $\theta_2 = \frac{Ml}{3EI}$	$y = \frac{Mlx}{6EI}\left(1 - \frac{x^2}{l^2}\right)$	$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \frac{l}{\sqrt{3}}$ $\delta = \frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \frac{7\omega_o l^3}{360EI}$ $\theta_2 = \frac{\omega_o l^3}{45EI}$	$y = \frac{\omega_o x}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = 0.00652\frac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\frac{\omega_o l^4}{EI}$ at the center

Summary of Essential Concepts of Theory of Structures

2.1 Single Formula Concept and its Applications

The most significant and well known formula in theory of structures is about the bending of a beam.
Now, consider a simple beam AB as shown below. If a moment "M" is applied at one end A, how the beam will behave or deform is all we need to know.



The bending of the beam or the rotations at the ends will obviously depend upon the following three factors.

i. Applied Moment = M (directly proportional)
ii. Length of the beam = L (directly proportional)
iii. Flexural Rigidity = EI (inversely proportional)

The true equation of above quantities is given below:

$$\Theta_A = ML/3EI\ , \qquad \text{----------(1)}$$

$$\Theta_B = ML/6EI \qquad \text{----------(2)}$$

Proof:
For the given beam, corresponding conjugate beam is also drawn.

Total load on the conjugate beam is ML/2EI.

Therefore $V_A^C = 2/3(ML/2EI)$ and $V_B^C = 1/3(ML/2E)$

$\Theta_A = V_A^C = ML/3EI$ and $\Theta_B = V_B^c = ML/6EI$

It is Suggested that the Formula derived above.

i.e $\Theta_A = ML/3EI$ & $\Theta_B = ML/6EI$ may be Considered as a **Single Important Formula** which will have almost all the applications.

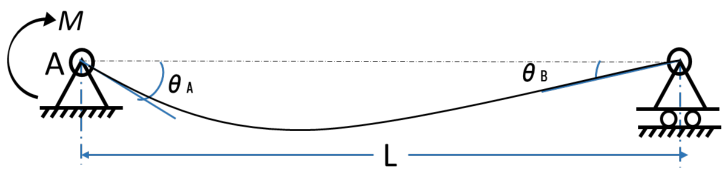
Above formula will be used in

(i) Defining stiffness of the beam
(ii) Calculation of end moments due to rotational weakness & sinking of Supports.
(iii) Calculation of Fixed end moments due to Loads
(iv) Derivation of Slop-deflection equations
(v) Derivation of Theorem of Three Moments.
(vi) Distribution Factors in Moment Distribution Method and Kani's Method
(vii) Problems involving Symmetry and Antisymmetry

2.2 What is Stiffness of a Beam

The Single important formula states that

$$\Theta_A = ML/3EI = \Theta$$



<u>Stiffness (or Stiffness Factor) is defined</u> to be the Moment required to produce unit rotation i.e. Θ_A or $\Theta = 1$

Therefore, <u>Stiffness of the Beam AB</u> is [M= 3EI/L] Where end B is hinged.

2.3 Four cases of Stiffness of the Beam

(i) When far end B is hinged (Above Case) Stiffness ($M/\Theta_A = 3EI/L$) -----(1)

(ii) When far end B is Fixed i.e. $\Theta_B = 0$

Since $\Theta_B = 1/2\ \Theta_A$.

When you make $\Theta_B = 0$,

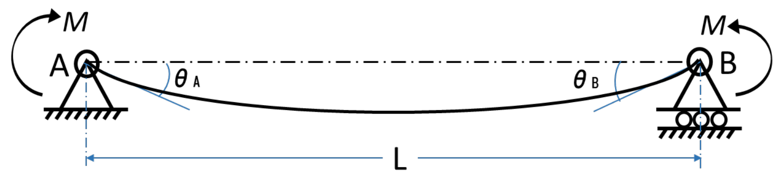
Revised value of $\Theta_A = ML/3EI - 1/2\ (1/2\Theta_A) = ML/3EI - ML/12EI = ML/4EI$

Therefore, Stiffness = $M/\Theta_A = 4EI/L$ --------(2)

This is Called <u>Standard Stiffness</u>

(iii) **A Case of Symmetry**

When the moments applied at A & B produce equal rotation i.e. $\Theta_A = \Theta_B$



and the bending of the beam is symmetrical about the central line, we call it a case of symmetry.

When only moment M is applied at A, Θ_A= ML/3EI (Case-1)

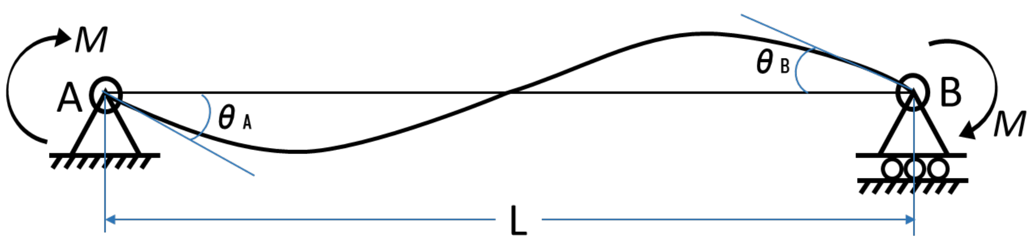
Now add Moment at B, this will increase the rotation at A by an amount 1/2 ML/3EI or ML/6EI

Revised value of Θ_A= ML/3EI + Ml/6EI = ML/2EI

Therefore Stiffness of the beam = M/Θ_A = 2EI/L ---→(3)

(iv) **A Case of Anti-symmetry**

When the moments applied at A & B produce equal rotation in same clockwise direction as shown in figure.



i.e. $\Theta_A = \Theta_B$ and bending of the beam is symmetrical but in opposite direction (this is known as Antisymmetry)

When only moment M is applied at A (case-1 above), Θ_A = ML/3EI

Now add(or apply) moment M at B in the same clockwise direction, this will decrease the rotation at A by an amount½ML/3EI or ML/6EI

Revised value of Θ_A = ML/3EI - ML/6EI = ML/6EI

Therefore Stiffness of the beam = M/Θ_A = 6EI/L -------(4)

2.4 Comparison or Relative Stiffnesses

i)	When far end fixed	**4EI/L**	**=**	**Standard stiffness**
ii)	When far end hinged	3EI/L	=	¾ (4EI/L) **= ¾** Standard stiffness
iii)	A case of Symmetry	2EI/L	=	½ (4EI/L) **= ½** Standard stiffness
iv)	A case of Anti Symmetry	6EI/L	=	3/2 (4EI/L) **= 3/2** Standard stiffness

Explanations for Symmetry and Anti-Symmetry of a frame:

Symmetry: A case of symmetry occurs when the given frame has a centroidal axis of symmetry with similar loadings on each side of the axis producing exactly the same bending moment diagrams on either side of the axis.

Anti-Symmetry: A case of anti-symmetry occurs when the given frame has a centroidal axis of symmetry with opposite loadings on either side of the axis, so that, bending moment diagrams will be in opposite directions with same magnitude.

Note: Please note that if a frame has got an axis of symmetry doesn't mean that it will be a case of symmetry or anti-symmetry. In fact it will all depend upon the magnitude and geometry of the loading on the either sides of the axis.

2.5 Stiffness of Joint

It is defined to be the sum of stiffness of all the members meeting at the joint. Suppose there are 4 members meeting at a joint with 2 members with hinged end and 2 with fixed ends,

Stiffness of the joint $= 4EI_1/L_1 + 4EI_2/L_2 + ¾.(4EI_3/L_3) + ¾.(4EI_4/L_4)$

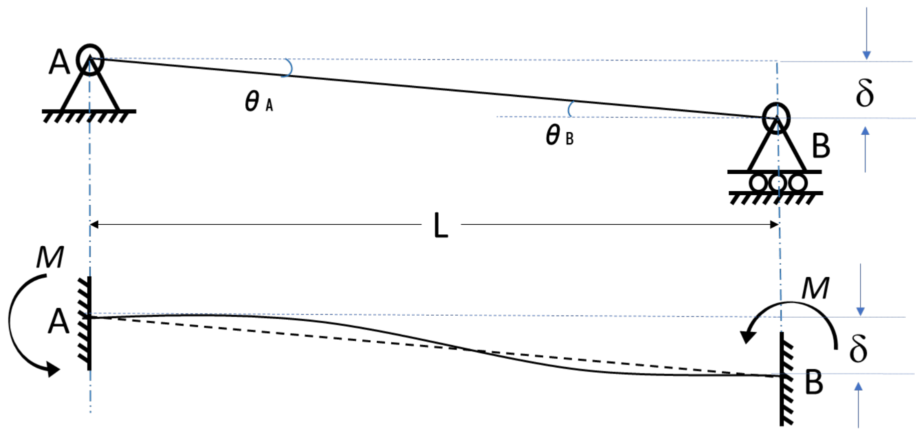
Distribution factor = Individual stiffness of the member / Total stiffness of the joint

Distribution factor of Member 1 $= (4EI_1/L_1) /\{4EI_1/L_1 + 4EI_2/L_2 + ¾.(4EI_3/L_3) + ¾.(4EI_4/L_4)\}$

Chapter 3 : Applications of Single Formula Concept

3.1 Sinking of supports:

(clockwise end moments will be assumed +ve)



Consider a beam AB in which, Supports B sinks or settles by amount "δ" .No bending of beam will occur, but the straight beam will rotate at A and B by

$\Theta_A = \Theta_B = \delta/L$

Now apply the end moments in anticlockwise (↺) direction to make the rotations Θ_A and $\Theta_B = 0$

Both these moments will be equal and anticlockwise.

When moment at A is applied and no moment at B:

Θ_{A1} = - ML/3EI (-ve because of anticlockwise moment)

When no moment is applied at A and moment M is applied at B:

Θ_{A2} = + ML/6EI (half of at B),

Therefore, $\Theta_A = \Theta_{A1} + \Theta_{A2} = - ML/6EI$,

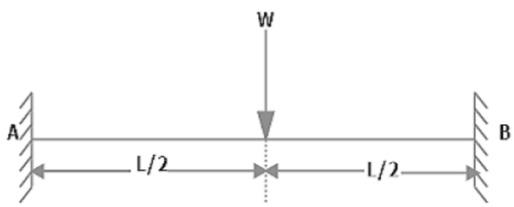
And also, $\Theta_A = \delta/L$

Therefore, $M = - 6EI\delta / L^2$

3.2 Calculation of Fixed End Moments due to loads

Simple problems

i) A concentrated load W at the centre



For a simply supported beam AB,

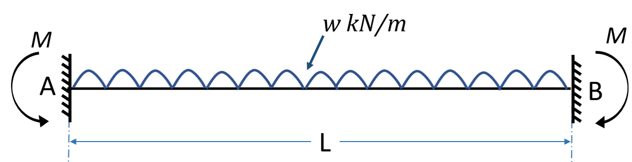
$\Theta_A = \Theta_B = WL^2/16EI$ (already known).

In order to make Θ_A and Θ_B zero, equal moments have to be applied at A & B say M.

Net value of $\Theta_A = WL^2/16EI = -ML/3EI + ML/6EI = -ML/2EI$

Therefore, $M=-WL/8$, $M^F_{AB} = -WL/8$ & $M^F_{BA} = -WL/8$

ii) Uniformly distributed load *wkN/m*



Refer previous pages in which

$\Theta_A = \Theta_B = wL^3/24EI$ for a simple beam

When beam is fixed at both ends, by applying moments at A & B, Θ_A & Θ_B are made equal to Zero.

Due to M at A, $\Theta_{A1} = -ML/3EI$

Due to M at B, $\Theta_{A2} = + ML/6EI$ (i.e. half of Θ_{A1})

Therefore, $wL^3/24EI = -ML/3EI + ML/6EI = - ML/2EI$

Hence, $M = - wL^2/12$

3.3 Derivation of Slope-Deflection Equations:

Consider a beam AB, in which final moments M_{AB} and M_{BA} are to be calculated.

Suppose it is given that fixed end moments are M^F_{AB} and M^F_{BA}

Also Θ_A, Θ_B and support settlement δ is given.

We will have M_{AB} equal to 4 quantities added together

i) M^F_{AB}

ii) Moment due to Θ_A, keeping $\Theta_B = 0$

iii) Moment due to Θ_B, keeping $\Theta_A = 0$

iv) Moment due to settlement of "δ" at B

Hence, $\mathbf{M_{AB}} = M^F_{AB} + (4EI/L)\,\Theta_A + (2EI/L)\,\Theta_B - 6EI\delta/L^2$

$= M^F_{AB} + (2EI/L)(2\Theta_A + \Theta_B - 3\delta/L)$ ------→ (1)

Similarly, $\mathbf{M_{BA}} = M^F_{BA} + (2EI/L)(\Theta_A + 2\Theta_B - 3\delta/L)$ ------→ (2)

Slope deflection equations (1) and (2)

3.4 Derivation of Theorem of Three Moments:

The proof of Theorem of Three moments will be given with the help of single important formula :

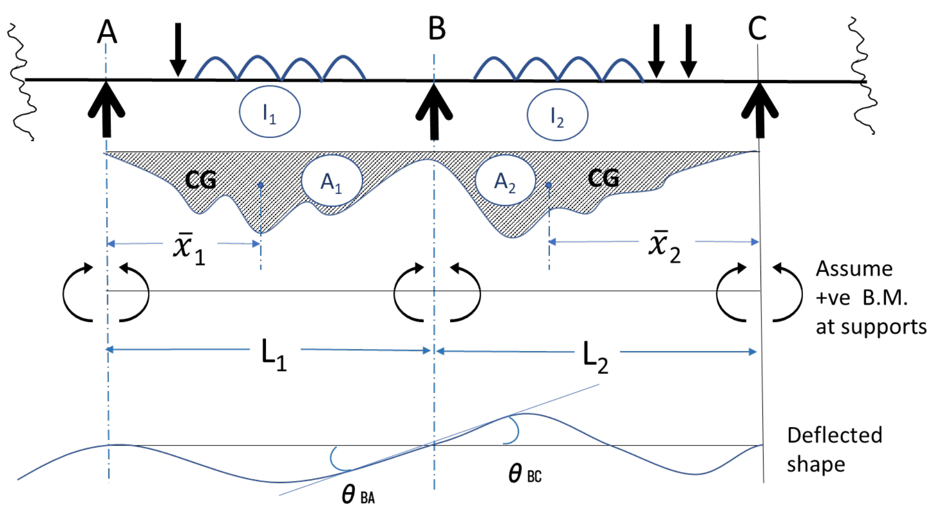
$$\theta = ML/3EI$$

Three unknown moments are M_A, M_B & M_C as shown in the diagram below & assumed to be +ve:

θ_{BA} will comprise of three quantities:

i) Contribution due to rotation at A
ii) Contribution due to rotation at B
iii) Contribution due to loads on AB

Therefore $\theta_{BA} = M_A L_1/6EI_1 + M_B L_1/3EI_1 + A_1 . \bar{x}_1 / EI_1 L_1$ ----→ (1)
(all in anticlockwise direction)



i) A1 and A2 are the areas of BMD for AB and BC due to loads only.
ii) Rotation at B for AB by conjugate beam method will be $= A_1 \bar{x}_1 / L_1$

Similarly $\theta_{BC} = M_C L_2/6EI_2 + M_B L_2/3EI_2 + A_2 . \bar{x}_2 / EI_2 L_2$ -----→ (2)
(all in clockwise direction for span BC at B)

Because of continuity tangent rotates in clockwise or anticlockwise direction,

Therefore $\theta_{BA} = - \theta_{BC}$

Hence $M_A L_1/6EI_1 + M_B L_1/3EI_1 + A_1 . \bar{x}_1 / EI_1 L_1 = - M_C L_2/6EI_2 - M_B L_2/3EI_2 - A_2 . \bar{x}_2 / EI_2 L_2$

Multiply by 6E, we get

$$\mathbf{M_A L_1/I_1 + 2M_B (L_1/I_1 + L_2/I_2) + M_C L_2/I_2 = - 6A_1.\bar{x}_1 / I_1 L_1 - 6A_2.\bar{x}_2 / I_2 L_2}$$

In case of settlement of supports say δ_A, δ_B, δ_C ; $(\delta_B - \delta_A)/L$ with -ve sign (when $\delta_B > \delta_A$) will be added in (1) above and similar correction will be made in (2) as well.

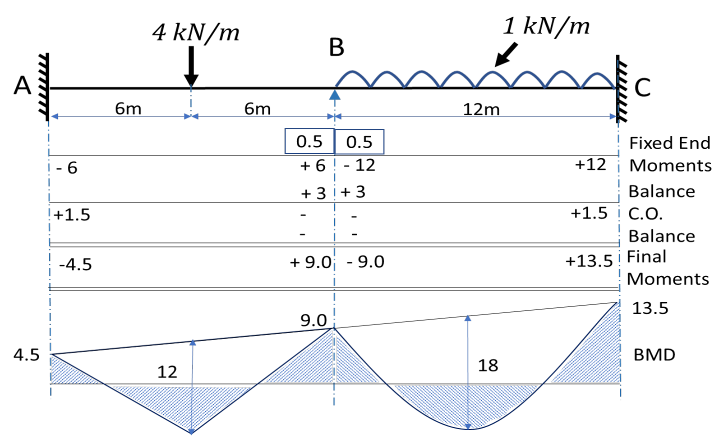
3.5 Application to problems of Moment Distribution:

Moment distribution method consists, in the beginning, to fix both ends of every member. In this manner we will have all fixed joints. In case, some of the ends are given to be hinged, these ends have to be modified by applying external moments at this end and half of the external moment is carried over to the other end. This fact has to be shown in the stiffness of the joint for the calculation of distribution factor as given above. In order to maintain the continuity of the remaining joints, external moments are applied at every joint and this is distributed to the members depending upon the their stiffness (distribution factor). This process is known as balancing. After each balancing the moment is carried to the fixed ends. The process continues till the balancing moments are small enough to be ignored. Nine examples are given to understand the fundamentals.

It may be noted that clockwise end moments are +ve and anticlockwise end moments are -ve.

Example 1

For a two span beam loaded as shown, using moment distribution method, sketch BMD giving principal values. EI = Constant



Fixed end moments

M_{AB}^F = - WL/8 = - (4x12)/8 = -6 kNm

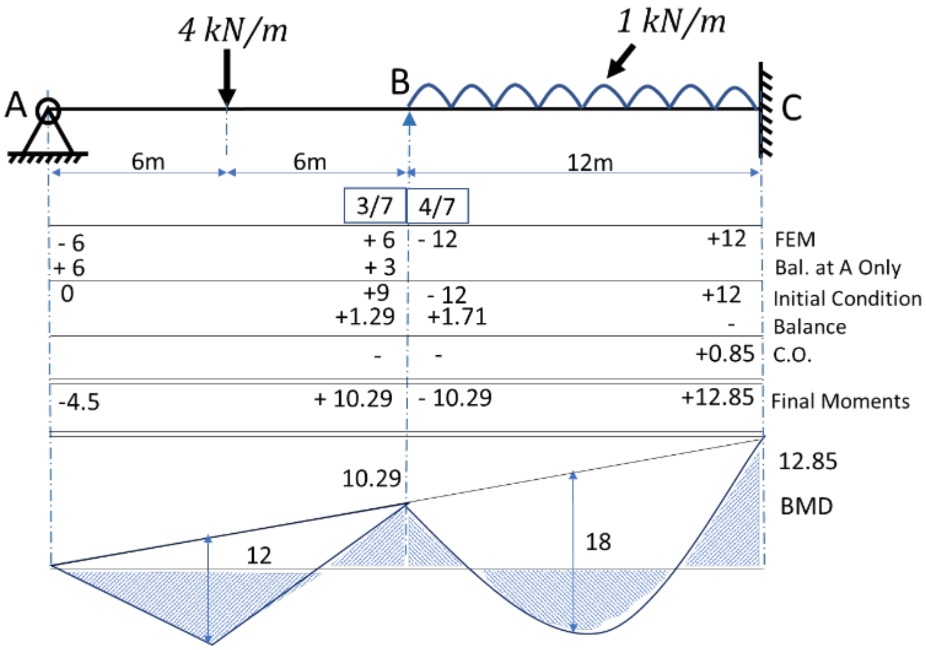
M_{BA}^F = + WL/8 = + (4x12)/8 = +6 kNm

M_{BC}^F = $wL^2/8$ = M_{CB}^F = (1 x 12^2)/12 = 12kNm

Clockwise End Moments +ve

Distribution Factors:			
Joint	Members	Stiffness	DF
B	BA	EI/12	1/2
	BC	EI/12	1/2

Example 2



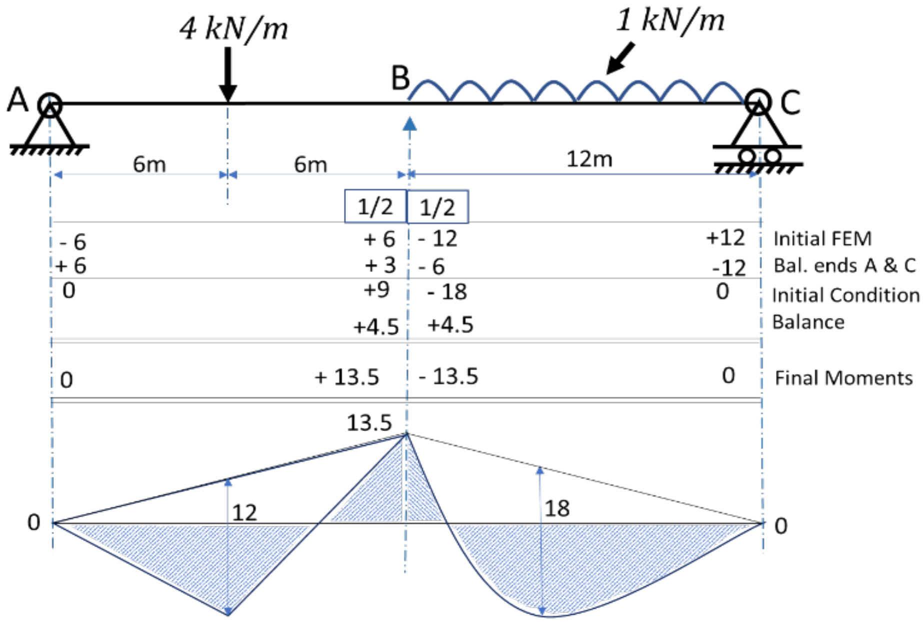
Solve above example keeping end A as hinged.

Fixed end moments will be same as above.

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/12).(**3/4**)	7	3/7
	BC	(EI/12).(4/4)		4/7

It may be noted that in column of relative stiffness we keep the denominator same, so that, sum columns show the sum of numerators only.

Example 3



Solve above example keeping end A and C as hinged ends.

Fixed end moments as calculated previously.

M_{AB}^{F} = - (WL)/8 = -6kN.m

M_{BA}^{F} = + (WL)/8 = +6kN.m

M_{BC}^{F} = - $(wL^2)/12$ = - 12kN.m

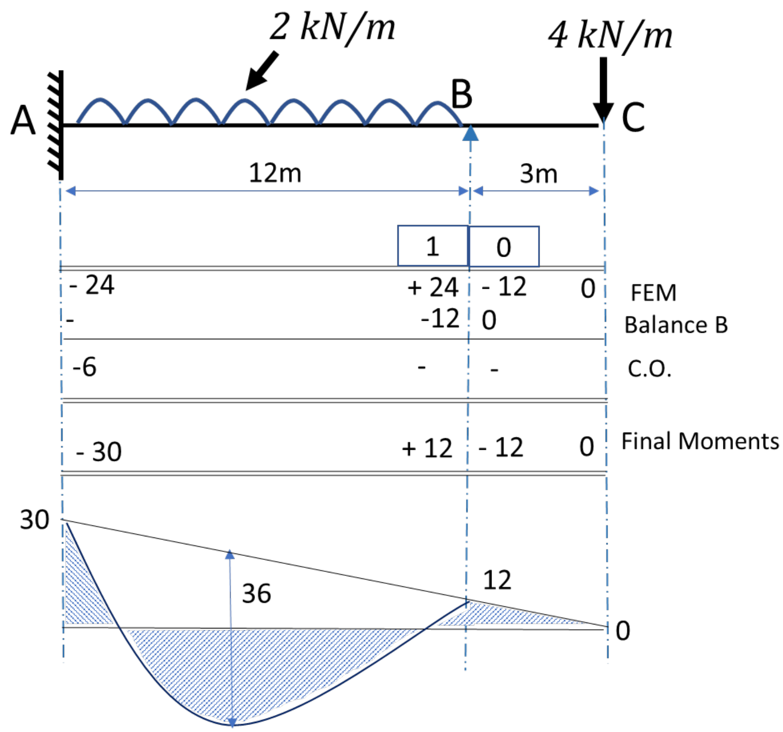
M_{CB}^{F} = +$(wL^2)/12$ = + 12kN.m

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/12).(**3/4**)	6	3/6 = ½
	BC	(EI/12).(**3/4**)		3/6 = 1/2

Notes:-

Above 3 examples: Students are advised to study the variation in bending moments because of change in one end and two end conditions and these should be as per their expectation in above three examples.

Example 4



BMD drawn on Tension Side

Keeping EI Constant & using moment distribution method, determine the bending moments for the beam loaded as shown and sketch BMD

M^F_{AB} = $-(wL^2)/12$ = -24 kN.m

M^F_{BA} = $+(wL^2)/12$ = + 24 kN.m

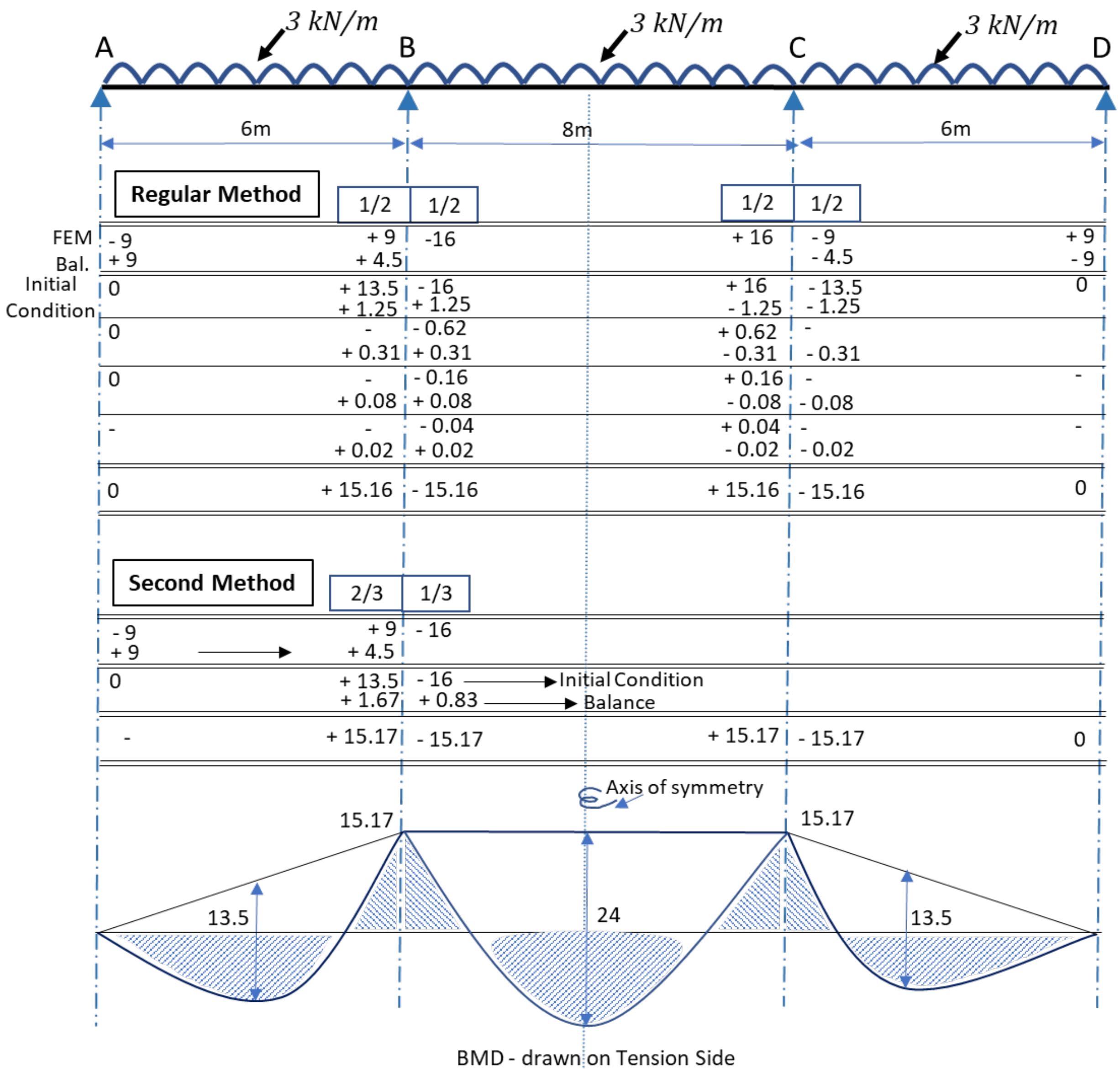
M^F_{BC} = -4×3 = - 12 kN.m

<u>Distribution factors:</u>

There is only one joint B where BA and BC meet. When balancing moment is applied BC will not absorb any moment since end C is free and nothing to hold it. Therefore **Distribution Factor(DF) for BC will be zero and for BA it will be one** because BA is the only member to absorb the moments.

Example 5

For a three-span continuous beam loaded as shown, determine the support moments and sketch the BMD giving principal values. EI = Constant



Fixed end moments for each span are calculated are shown below:

M^F_{AB} = - (wL^2)/12 =-3x6^2/12 = -9 kNm, M^F_{BA} = + (wL^2)/12 = 3x6^2/12 = +9 kNm

Similarly, M^F_{CD} = -9 kNm, M^F_{DC} = +9 kNm

M^F_{BC} = - (wL^2)/12 =-3x8^2/12 = -16 kNm, M^F_{CB} = + (wL^2)/12 = 3x8^2/12 = +16 kNm

i) <u>Regular Method</u>

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/6).(**3/4**)	2	1/2
	BC	(EI/8)		1/2
C	CB	(EI/8)	2	1/2
	CD	(EI/6).(**3/4**)		1/2

ii) <u>Second Method</u>

Taking advantage of symmetry – solve half the frame or beam

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/6).(**3/4**).(2/2)	3	2/3
	BC	(EI/8).(**1/2**).(2/2)		1/3

Explanation:

Since member BA is hinged at A, its stiffness is modified by ¾.

Since member BC is cut by axis of Symmetry its stiffness is modified by 1/2 as discussed previously.

Important Note

In Symmetry and Anti-symmetry problems, we get solution for half of the beam or frame**. In both the cases numerical values will be same for the other half also,** but the sign for Anti-symmetry will be same and for Symmetry opposite signs will be valid.

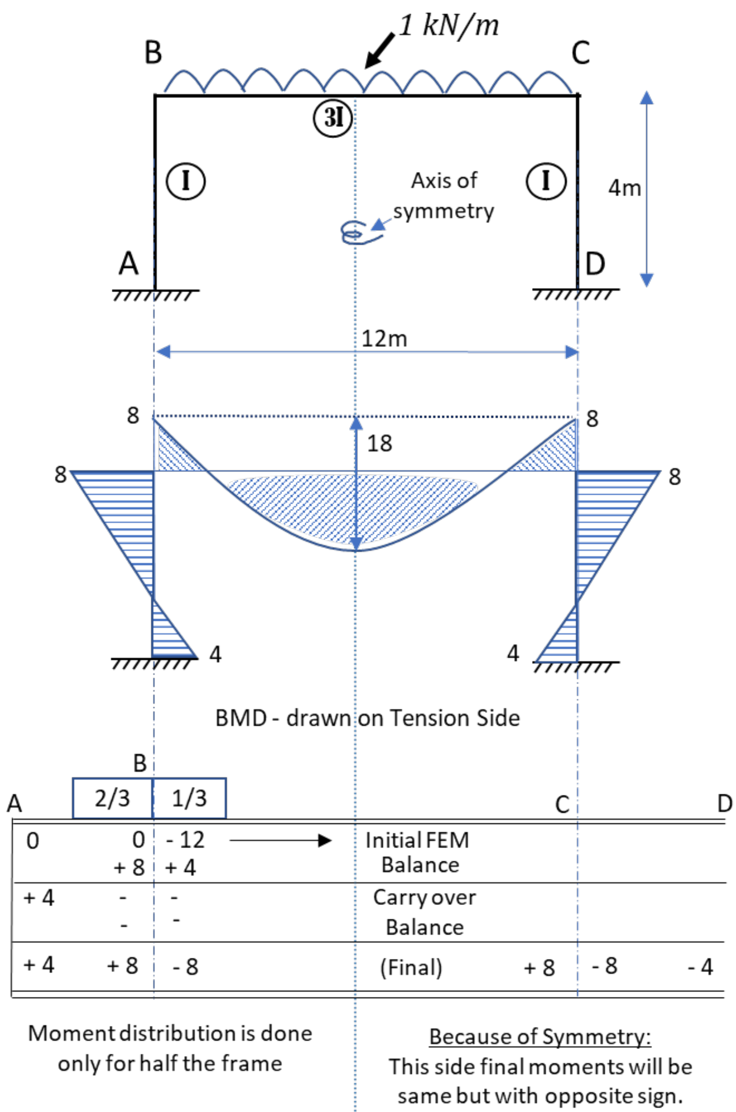
Example 6

Taking the advantage of symmetry analyze the given portal frame loaded as shown.

<u>Fixed end moments:</u>

M^F_{BC} = $-(wL^2)/12$ = -12 kN.m

M^F_{CB} = $+(wL^2)/12$ = $+12$ kN.m



Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/4).(2/2)	3	2/3
	BC	(3EI/12).(**1/2**)		1/3

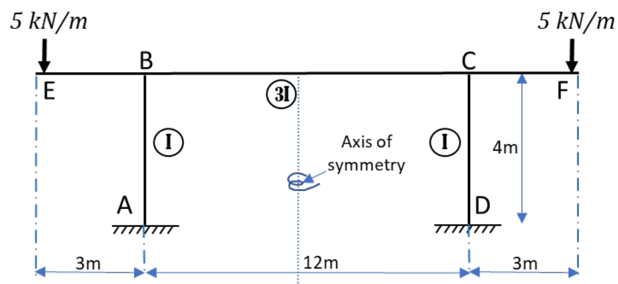
Example 7

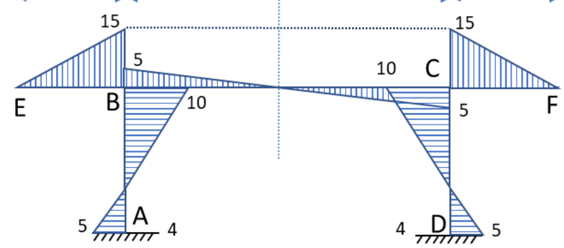
Taking the advantage of symmetry analyze the given portal frame loaded as shown. EI=Constant.

Fixed end moments:

No loads on BA & BC therefore, M^F_{BA} & M^F_{BC} are both zero

M^F_{BE} = + 15kN.m



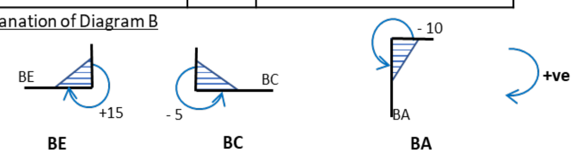


BMD - drawn on Tension Side

Note: - Hatching of the diagram should be perpendicular to the member concerned

		2/3	0	1/3	
	AB	BA	BE	BC	Solve half the frame
	0	0 - 10	+15 0	0 - 5	FEM Balance
	- 5 -	- -	- -	- -	Carry over Balance
	-5	- 10	+15	- 5	Final Moments

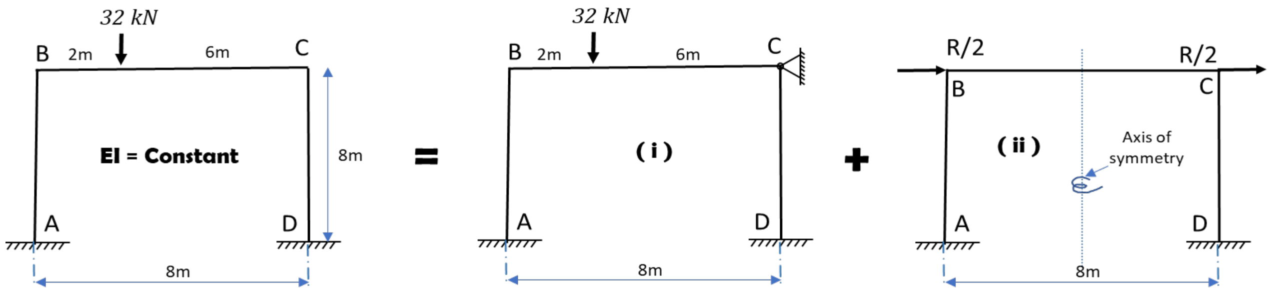
Explanation of Diagram B



Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BE	0	3	0
	BA	(EI/4).(2/2) = 2EI/8		2/3
	BC	(EII/12).(**1/2**)= EI/8		1/3

Example 8

For the portal frame loaded as shown, sketch BMD giving values of principal moment



Given Frame **Non Sway Frame (i)** **Sway Frame (ii)**

First method (So called regular method)

Since the lengths of all the members is 8m and EI is also same. At B and C, the distribution factors will be ½ and ½

i) <u>Non Sway Moment Distribution:</u>

$M^{F}_{BC} = -(32 \times 2 \times 6^2)/8^2 = -36 kNm$

$M^{F}_{CB} = +(32 \times 2^2 \times 6)/8^2 = +12 kNm$

A	B 1/2	1/2	C 1/2	1/2	D	
0 -	0 - 18	- 36 + 18	+ 12 - 6	0 - 6	0	Initial FEM Balance
+9 -	- + 1.5	-3 -1.5	+9 -4.5	- - 4.5	-3 -	C.O Bal
+0.75	- +1.13	-2.25 + 1.12	+0.75 -0.38	- - 0.37	-2.25 -	C.O. Bal
+0.56 -	- + 0.09	-0.18 +0.09	+0.56 -0.28	- - 0.28	+0.18 -	C.O Bal
+0.04 +0.03	- +0.07	-0.14 +0.07	+0.04 -0.02	- - 0.0 →	-0.14 -0.01	C.O (Last Bal Cycle)
+10.38	**+20.79**	**-20.79**	**+11.17**	**-11.17**	**-5.58**	Final Moments

Note: - At the fixed end, no balancing is done. It should be allowed to remain fixed. It will simply absorb half the moment applied at other end.

ii) <u>Sway Moment Calculations:</u>

In the (i) above the frame was prevented from swaying to right by applying force R (←) to the left at the beam level. This force R is to be removed by applying Force R/2 and Force R/2 (→) as sown above to permit sway action.

If you open the frame (ii), you will find that it is a case of Anti-Symmetry, and therefore, half the frame will be solved.

If "δ" is the displacement of the frame at the beam level (so called sway displacement) the initial fixed end moments will be – $6EI\delta/L^2$ for ends A,B, C and D.

Since "δ" the displacement of the beam to the right is not known we assume – $6EI\delta/L^2 = -20$.

Accordingly M^F_{AB} = M^F_{BA} = - 20 kN.m

Taking the advantage of anti-symmetry, the distribution factors will be as follows.

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Modified Stiffness	Sum	DF
B	BA	(EI/8) x (2/2)	5	2/5
	BC	(EI/8) x **(3/2)** Anti-symmetry		3/5

From the results of moments as done on the right.

Assuming value of R to cause an assumed sway "δ" will be equal to

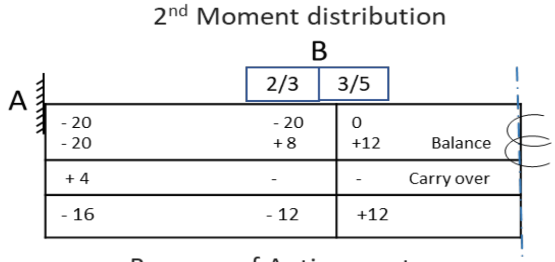
R' = (-16-12)/8 + (-16-12)/8 = -7 kN

True value of R (Obtained from 1st Moment distribution) is

R = (10.38 + 20.79)/8 + (- 11.17 – 5.58)/8 = 1.803 kN

Correction Factor for 2nd Moment Distribution = 1.803/7 = 0.2575

2nd Moment distribution



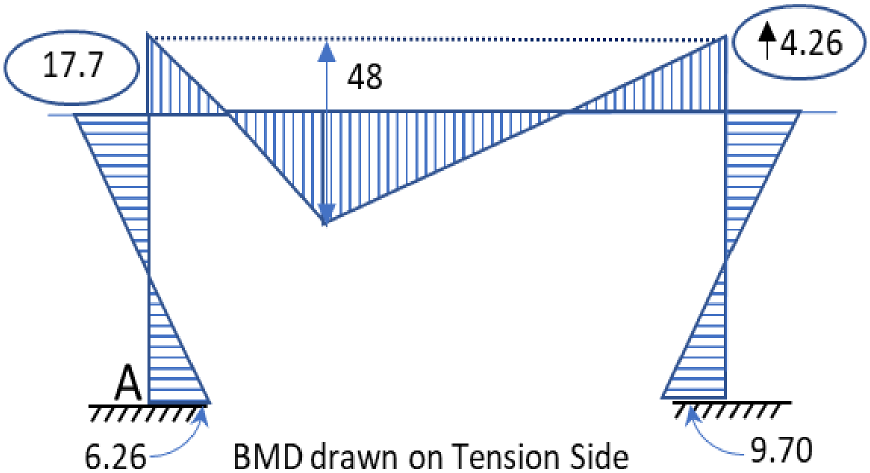
Because of Antisymmetry

$M_{CD}^F = 12$ & $M_{DC}^F = -16$

Final moments for 2nd MD,

M_{AB} = -16 x 0.2575 = -4.12 kNm

M_{BA} = - 12 x 0.2575 = -3.09 kNm



Final Moments for (i) & (ii) will therefore be as follows:

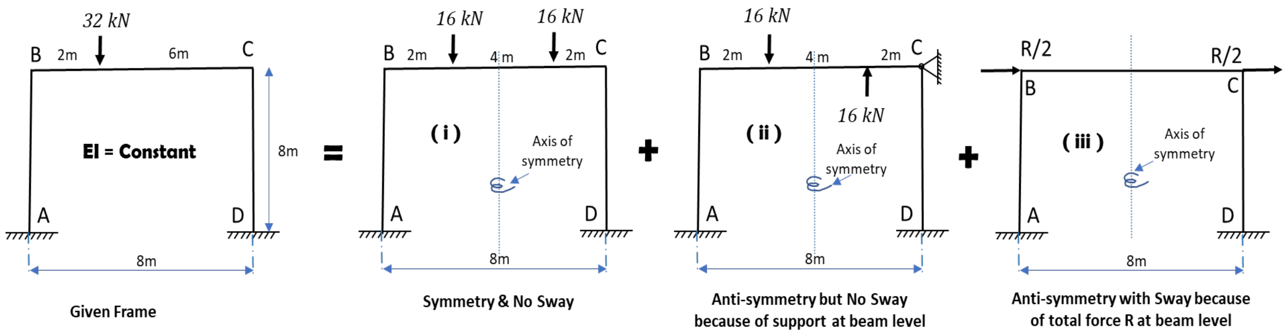
A		B		C	D	
+10.38 - 4.12	+20.79 - 3.09	- 20.79 + 3.09	+11.17 + 3.09	- 11.17 -3.09	-5.58 - 4.12	1st MD 2nd MD Corrected
+ 6.26	**+ 17.7**	**- 17.7**	**+14.26**	**-14.26**	**- 9.70**	Final

Example 9

Taking the advantage of symmetry and anti-symmetry, solve the previous problem (Example-8)

Solution:

The given portal frame is symmetrical, but the loading is not symmetrical. As such, the given problem will be solved by breaking it into 3 problems as shown below:

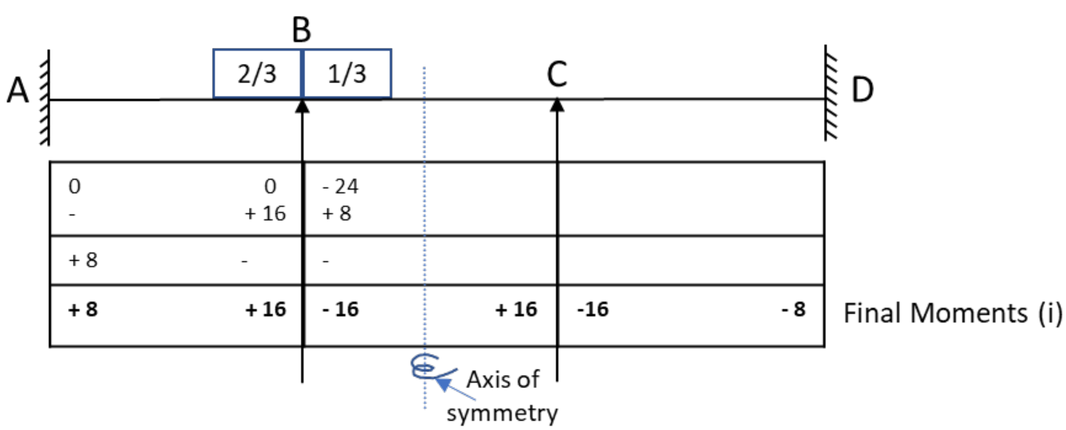


Solution for (i) above

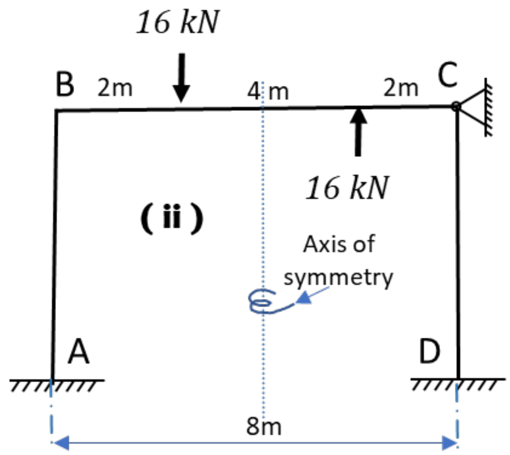
Fixed end moment $M^F_{BC} = (16 \times 6^2 \times 2)/8^2 + (16 \times 6 \times 2^2)/8^2 = 18 + 6 = 24$ kN.m

(Half the frame will be solved)

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/8) x (2/2)	3	2/3
	BC	(EI/8) x (**1/2**)		1/3



Solution for (ii)



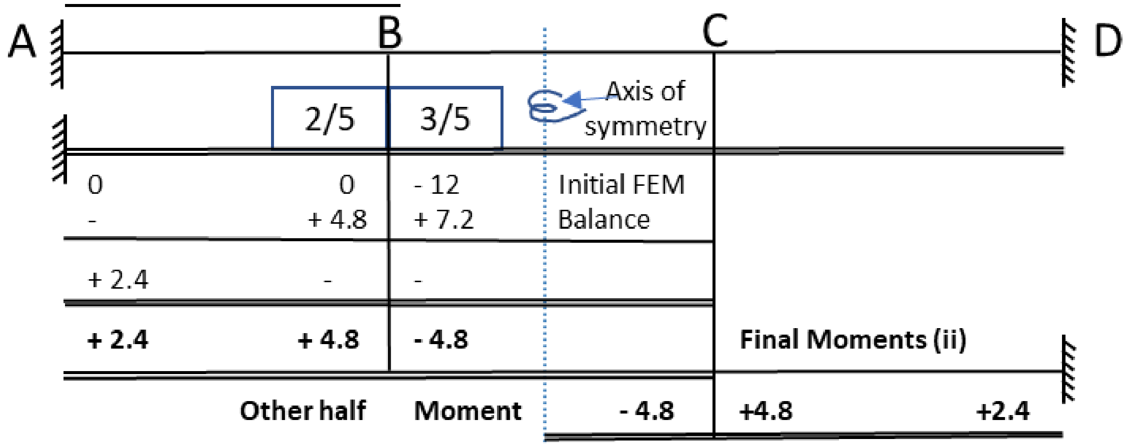
Anti-symmetry but No Sway because of support at beam level

M^F_{BC} = (16 x 6^2 x 2)/ 8^2- (16 x 6 x 2^2)/ 8^2 = 18 - 6 =12kN.m

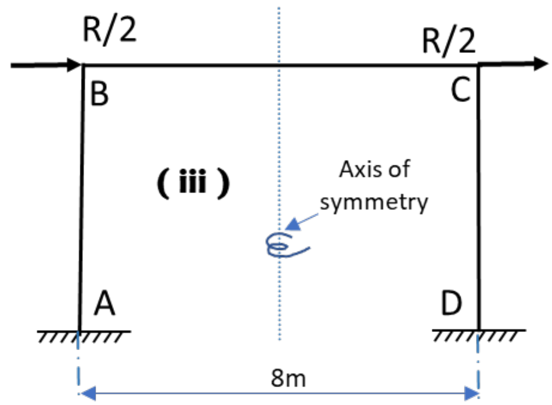
Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/8) x (2/2)	5	2/5
	BC	(EI/8) x (**3/2**)		3/5

Value of force R = (+4.8 + 2.3)/8 + (+4.8 + 2.3)/8 = 1.80 kN

Moment distribution



Solution for (iii)



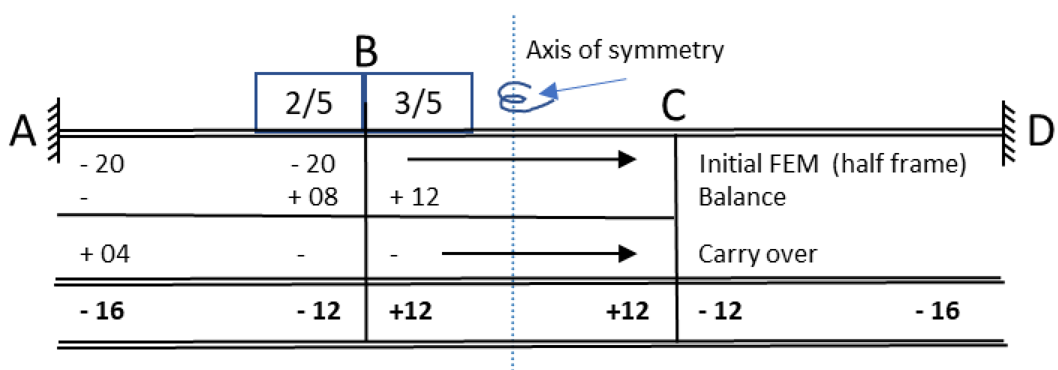
Anti-symmetry with Sway because of total force R at beam level

Distribution factors will remain same as given above i.e. for BA 2/5 and for BC 3/5

Due to loading as shown, there will be horizontal sway at BC level say "δ"

Therefore Fixed End Moments will be $-6EI\delta/L^2$for AB, BA, CB & CD. But we will solve the frame.

Assume $-6EI\delta/L^2$= -20, Now Moment Distribution



Since this is a case of Anti-symmetry, on right half frame same values of BM with same signs will be valid.

Above calculations will give assumed value of R

R (assumed) = (-12-16) / 8 + (-12-16) / 8 = -7kN (←)

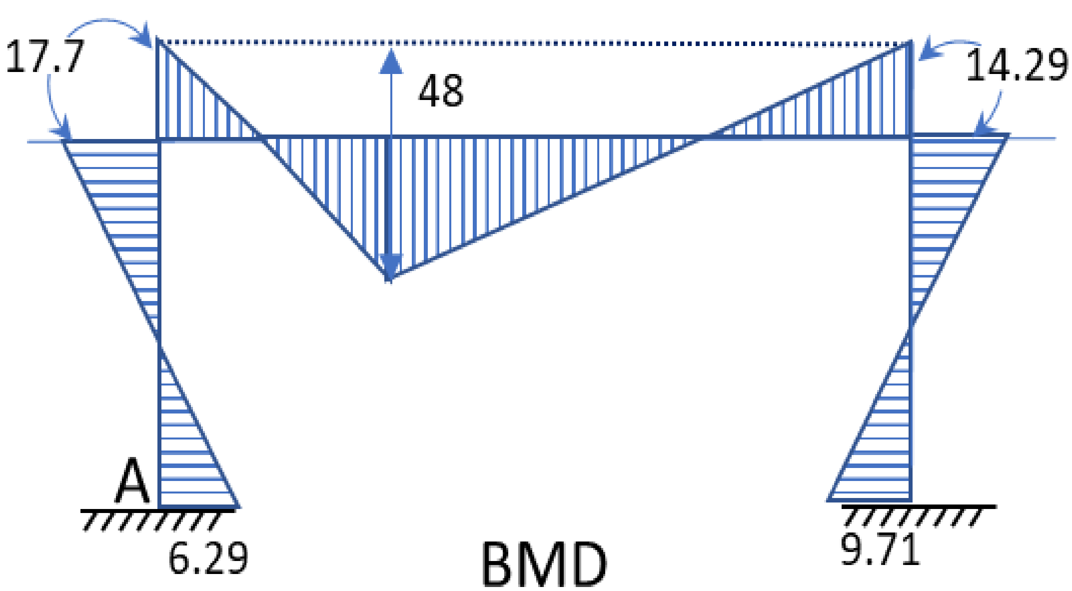
True value of R is 1.8 kN (←)

Therefore correction factor for "Bending Moments" in (iii) will be (1.8 / 7) And corrected moments will be as follows:

AB	BA	BC	CB	CD	DC	
A		B		C		D
- 16	-12	+ 12	+12	- 12	- 16	Assuming Δ
Multiply by 1.8/7						
- 4.11	**- 3.09**	**+ 3.09**	**+ 3.09**	**- 3.09**	**- 4.11**	Correct value (iii)

Final Moment (i) + (ii) + (iii)

AB	BA	BC	CB	CD	DC	
A		B		C		D
+ 8	+ 16	- 16	+ 16	-16	- 8	(i)
+ 2.4	+ 4.8	- 4.8	- 4.8	+4.8	+2.4	(ii)
- 4.11	- 3.09	+ 3.09	+ 3.09	- 3.09	-4.11	(iii)
- 6.29	**+17.71**	**-17.71**	**+14.29**	**-14.29**	**-9.71**	Final Moments (i) + (ii) + (iii)



Chapter 4 : Box Frames and their Design Considerations

4.1 Design considerations for an R.C.C Box Culvert

Underground structures are used for some specific purposes. These are normally used for carrying water. At other times they are uses for grade crossing for pedestrians and cycle traffic. These are also known as buried structures/ The most prevalent are pipes and box culverts.

Buried structures with horizontal dimensions less than 10ft or 3m are not classified as bridges. These small box culverts do not require extensive design and are selected from the standard design tables. Buried structures with horizontal dimensions greater than 10ft are considered as bridges and therefore are designed as per bridge rules of the concerned country.

Underground structures carry vertical loads through a combination through a combination of internal capacity and soil arching around the structure. Concrete box culverts and rigid pipes are classified as rigid culverts and are assumed to carry the design loads internally with limited requirement or benefit of the soil.

Where pipe solutions are inappropriate, box culverts are the default buried structure type. Their large openings are often required to provide adequate hydraulic capacity. Box culverts are also frequently used for pedestrians or cattle underpasses.

The reinforcement used in concrete box culverts can eb either conventional bar reinforcement or welding wire fabric. Welded wire fabric has a yield strength slightly larger than conventional bar reinforcement (65 Kgs vs 60 Kgs).

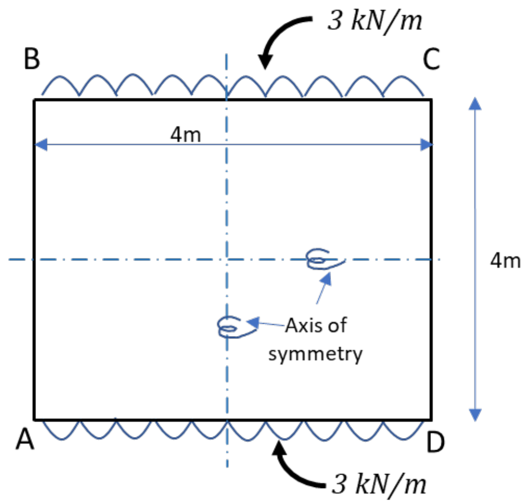
Standard designs for precast concrete box culverts are available with spans varying from 6 to 16 feet and rises varying from 4 to 14 feet.

Standard precast concrete box culverts are typically fabricated in 6 foot sections

4.2 Box Frames Examples

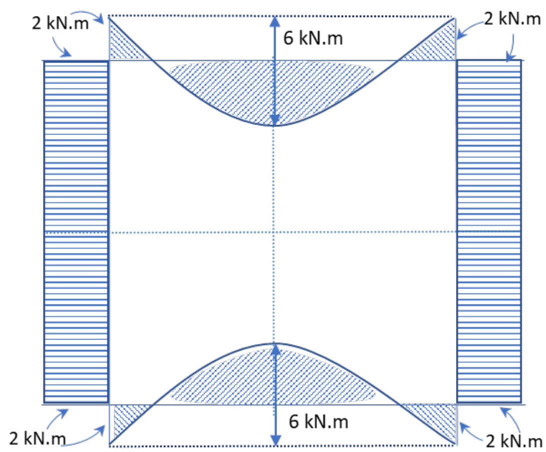
Example 1

Analyze the box frame loaded as shown below. Get BMD and principal values:



$M_{BA}^F = 0$, $M_{BC}^F = (-3 \times 4^2)/12 = -4$kN.m

There are two symmetrical axes and therefore we can take double the advantage of Symmetry



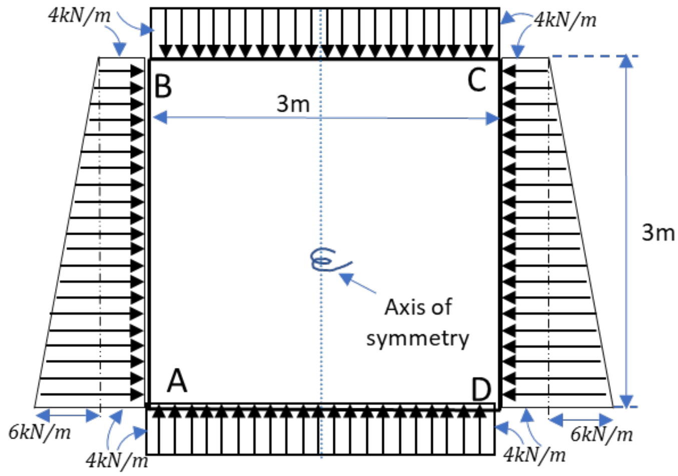
BMD - drawn on Tension Side

	BA	BC	
DF	1/2	1/2	
	0	- 4	→ Initial FEM
	+ 2	+2	→ Balance
	+ 2	**- 2**	**Final Moments**

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(I/4) x (**1/2**)	2	1/2
	BC	(I/4) x (**1/2**)		½

Example 2

Analyse the box frame loaded as shown (because of being buried under soil)
Draw BMD giving the principal values



FEM, M^F_{BC}

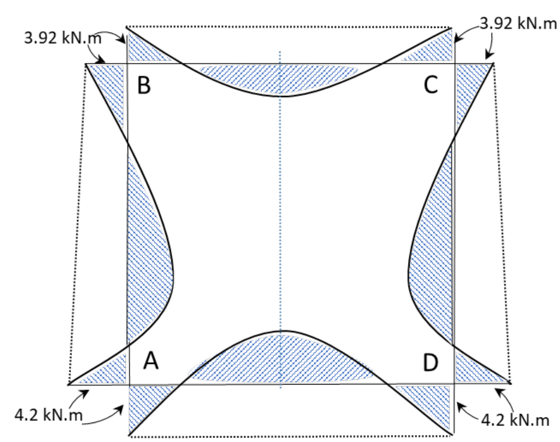
M^F_{BC} = $(-wL^2)/12$ = $(-4 \times 3^2)/12$ = -3 kN.m

M^F_{BA} = $(+wL^2)/12 + WL/15$ = $+(4 \times 3^2)/12 + ((6 \times 3)/2) \times (3/15)$ = +4.8 kN.m

M^F_{AB} = $(-wL^2)/12 - WL/10$ = $+(4 \times 3^2)/12 + ((6 \times 3)/2) \times (3/10)$ = - 5.7 kN.m

Distribution Factors (Bold Figures indicate modifying factor for stiffness)				
Joint	Members	Stiffness	Sum	DF
B	BA	(EI/3) x (2/2)	3	2/3
	BC	(EI/3) x (**1/2**)		1/3

AD	AB	BA	BC
1/3	2/3	2/3	1/3
+ 3.0 + 0.9	-5.7 +1.4	+4.8 -1.2	- 3.0 - 0.6
+ 0.2	+ 0.6 + 0.4	+ 0.7 - 0.47	- - 0.23
+0.08	- 0.24 + 0.16	+ 0.2 - 0.13	- 0.7
+0.02	- 0.07 + 0.05	+ 0.08 - 0.06	- 0.2
+ 4.2	**- 4.2**	**+ 3.92**	**- 3.92**



BMD - drawn on Tension Side

Chapter 5: Design of Steel Structures

In most of the universities, this paper is taught at 6th or 7th Semester level. The course content of this subject is Design of compression members, tension members, lattice girders, plate girders, built up beams, columns, riveted and welded connections.

The portion dealing with long span structures has neither been dealt at this level or in theory of structures. Some of the world heritage bridge structures have been built on the theory of analysis of long span structures.

Therefore, it is desirable to give brief design approach to solve such problems.

5.1 Long Span Structures (with special reference to Railway Bridge Trusses)

As the span of structure becomes large bending moment to which structure is subjected increases rapidly, if simply supported structure are used. Even if the load per meter square to be carried by the structure did not increase, the bending moment with the distributed loads would vary with the square of the span. Actually the dead weight increases with the span, so that the bending moment increases at a much greater rate.

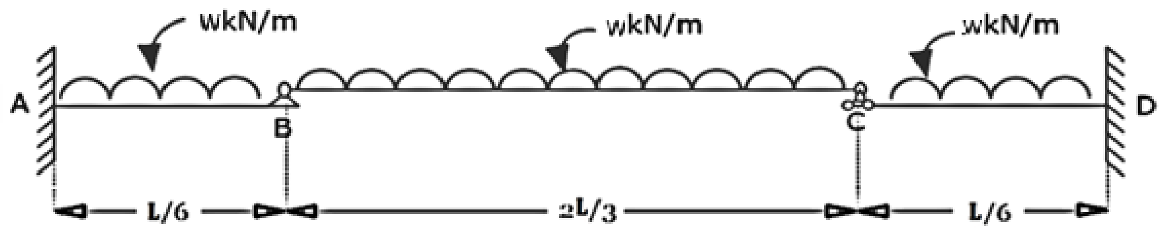
Since the chord Stresses in a truss depend upon the bending moments carried by the truss, these considerations are important not only from the point of view of the design of members of the truss, but also for design of stringers and cross girders.

For a structure to be economical, it is desirable for the case of long span to adopt some means of construction to reduce the bending moments. There are number of methods to do this. Here, a method will be given which will make the analysis simple. In order to make the structure determinate**, some special construction features will be added** so that the design becomes simple.

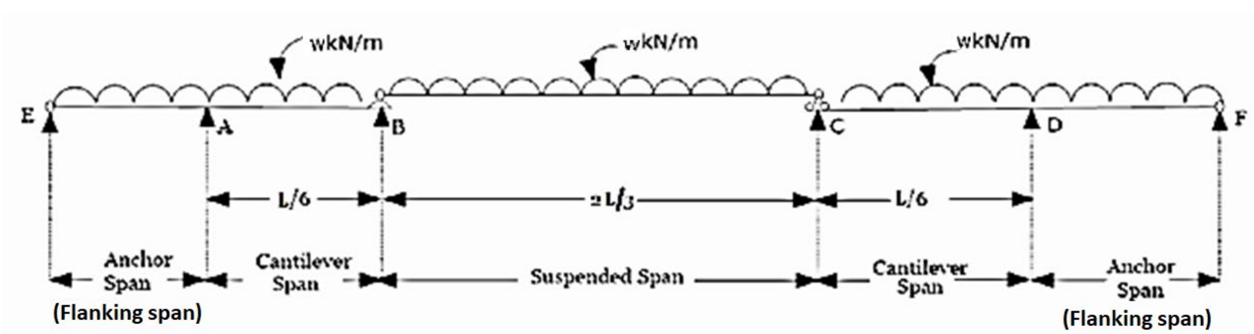
5.2 General Method

This is known as cantilever-girder system or a balanced-cantilever-bridge.

The total length "L" is divided into 3 parts. Main length of the beam is reduced to 2/3.l . Remaining two end portions of cantilevers "l/6" and "l/6" as shown below.



Maximum +ve B.M. in portion BC = $w(2/3 . L)^2/8 = w.L^2/18$
Maximum -ve B.M. in portion AB or CD is = $-w/2(L/6)^2 - (2/3. wL)L/2 . L/6$
$= -wL^2/2 - wL^2/18$
$= - 5/72\ wL^2$
The beam must therefore be designed for a B.M of $5/72wL^2$
The quantity $5/72.wL^2$ is about 55 % of $wL^2/8$
It means reduction of B.M. is nearly 45%
It may be noted that it is difficult to obtained fixity conditions at A and D i.e. cantilever end for AB & CD. Cantilever arms.
In order to resist the moments at A and D Flanking Span AE and DF have been created as shown in the figure.

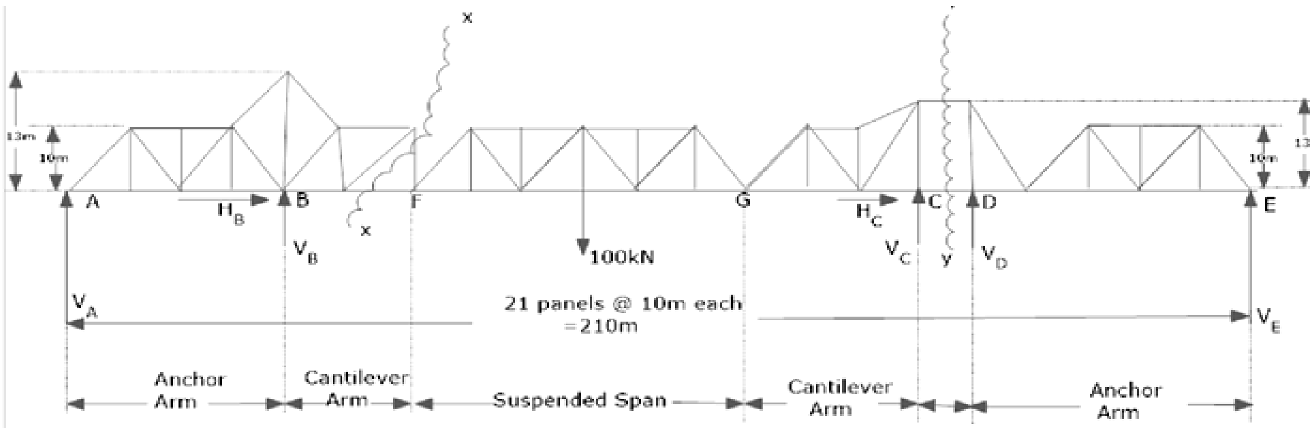


The desired moments at A and D are resisted by the Flanking spans AE and DF. If need be, additional **anchorages** can be provided at reaction E and F.

5.3 Statical Conditions in Balanced-Cantilever Bridges by providing "Special Construction Features"

Following example shows typical special construction features. In order to calculate the forces in the members, we must have the values of all the unknown reactions or in fact the influence lines for all the reactions.

The diagram of the truss given below show that there are **7 unknown reactions (5 Vertical and 2 Horizontal).**



Statical conditions of equilibrium provides 3 equations. We need to provide 4 more equations.

For this we require **4 special construction features**. These are
(i) Hinge "F" of the Suspended Span
(ii) Hinge "G" of the Suspended Span
(iii) Horizontal force cannot be transferred from left hand side of section x-x. (Theoretically no horizontal members)
(iv)Vertical force cannot be transferred from left hand side of section y-y to right hand side. (Theoretically no diagonal members)

Thus, we have,
(1) $\Sigma H = 0$ for all the forces on entire structure
(2) $\Sigma V = 0$ for all the forces on entire structure
(3) $\Sigma M = 0$ for all the forces about any point in the plane
(4) ΣMat F = 0 for all the forces one side of Hinge F
(5) ΣMat G = 0 for all the forces on one side of Hinge G
(6) $\Sigma H = 0$, for all the forces on one side of section x-x
(7) $\Sigma V = 0$, for all the forces on one side of section y-y.

In the above example, load of 100 kN is given at the center of suspended span. With this data, Reactions may be determined and therefore the forces in members can be calculated. This is just a problem for practice and the knowledge regarding the special construction features.

Chapter 6: FORTH BRIDGE, Scotland, UK

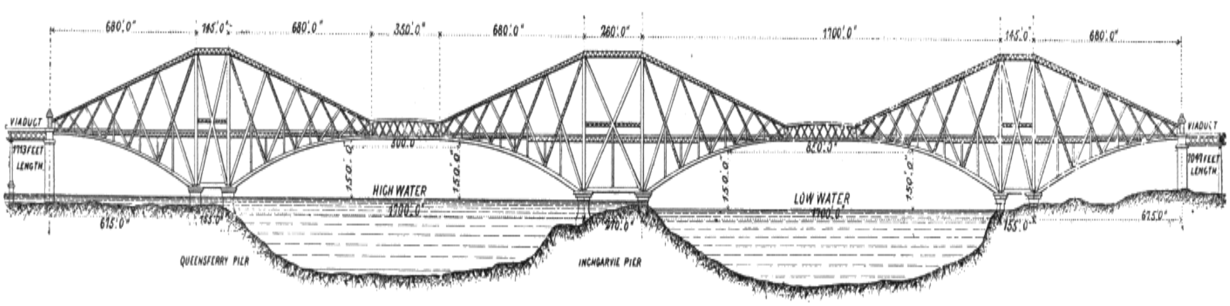
List of Longest Cantilever Bridge Spans :

The attempt is made to give only first six cantilever bridges.

S. No.	Name	Location	Span in Meters	Completed
1.	Pont de Quebec	Quebec, Canada	Span 549m	1917
2.	Forth Bridge	**Scotland, UK**	Span 521m	1890 (Two Spans)
3.	Minato Bridge	Osaka, Japan	Span 510m	1973
4.	Commodore Barry Bridge	New Jersey, USA	Span 501m	1974
5.	Crescent City Connection	New Orleans, USA	Span 480m	1958/1988
6.	Howrah Bridge	**Kolkata, India**	Span 457m	1943

<u>Forth Railway Bridge Scotland, U.K.</u>

This bridge is 2467m (8094 ft.) long. Nearly 2.5 km. It has two Cantilever spans of 521m (1750ft.) Unesco has declared this as World Heritage Center. It is called as Unesco World Heritage Center. It is a two track railway bridge.



Dimensions

(i) Length of the bridge is 2467m (8094 ft.)
(ii) Two main spans are 518.6m(1710 ft.)
(iii) Two side spans are 207.3m(680 ft.)
(iv) 15 approach spans of 51.2m (168 ft.)

Each main span consists of two cantilever arms 207.3m (680 ft.) supporting a central span of 106.7m (350 ft.).

The weight of the steel in the bridge is 51,320 tons including 6.5 million rivets. the bridge also used 640,000 cubic- feet of granite.

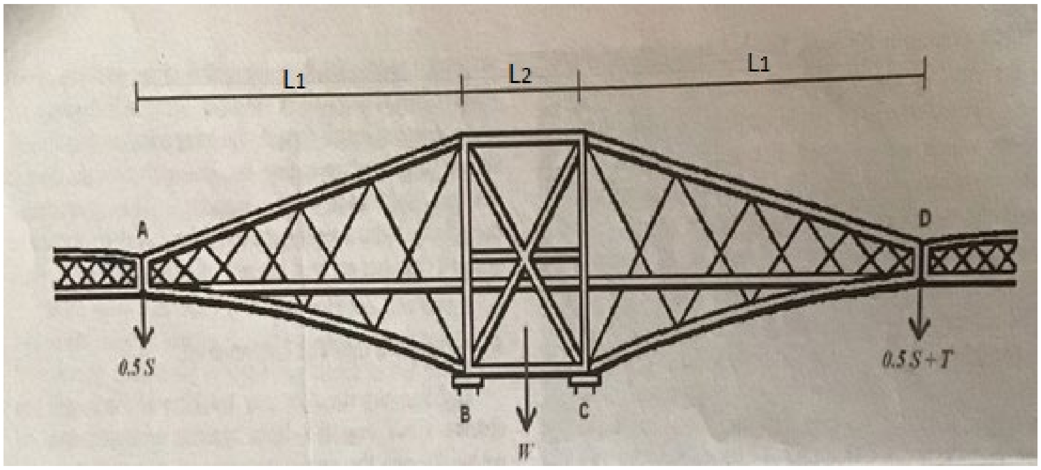
There are three towers supporting the cantilever arms. Each tower resting on separate granite piers.

The painting work on the whole Forth Railway Bridge was completed in December 2011 and is expected to last for **25 years**. 54,000 liters of paint use.

Chapter 7: Engineering Principles of Analysis & Design

7.1 Engineering Principles

The bridge is built on the Principle of the cantilever bridge where a cantilever beam supports a light central girder(so called suspended span). This principle has been used for several hundred years in the construction of bridges. This was demonstrated by the Engineers Fowler and Baker. There photo along with Japanese Engineers Koichi Watanabe is given in books and the technical Papers.



W = Dead weight of tower and cantilevers
S = Dead weight of Suspended Span
T = Train load assumed to act at a Point
L1 = Length of Cantilever
L2 = Width of tower

Mc = 0.5S.L1 + T.L1 ----① → Taking moments from Right hand Side

Mc = 0.5S(L1 + L2) + W. L2/2 ---② → Taking moments from left hand Side

7.2 Overturning

The "Worst Case" loading condition is, if two trains meet, travelling in opposite direction over one of the suspended spans, while there is strong wind blowing and the sun is blazing. Forgetting the effects of wind and temperature, the load of a train moving on the bridge immediately upsets the balance of the structure.

Now, Consider the above two equations, when the Train load is taken into account, Overturning takes place, when

$$0.5S\, L_1 + T.\, L_1 > 0.5S(L_1 + L_2) + \mathbf{WL_2}/2$$

i.e. $2\, T\, L_1 - SL_2 > W.L_2$ → ③ [Case of Over turning]

Prevention

In order to prevent overturning, width of the Tower i.e. "L_2" may be increased or Weight of the Tower "W" may be increased, or both.

7.3 Buckling Effect

Considering that the bridge carries huge compression forces, each of the struts must have been designed to resist buckling- most importantly large compression members of the cantilever arm.
The cross bracing between the top and the lowest strut members is in place to prevent the buckling of the steel tube and to support the heavy compression tube. A simple buckling calculation is necessary to ascertain how many bracing members are required or rather how frequently along the main strut they should be placed. Following assumption may be made.

1. Two trains moving over bridge meeting over suspended span with weight acting at a single point.
2. Compression members is one straight members (continuous) of constant section (12 ft = 3.66m)
3. Compression force is constant throughout the span.
4. At the point where cantilever meets the suspended span the compression members is at an angle of 12° to the horizontal.
5. Modules of elasticity steel E = 200kN/ mm^2.
6. Thickness of the member is one inch or 25.4 mm
7. Train dead load = 4000kN
8. Dead weight carried by the member = 80,000kN

$$\frac{\text{Compression in Member}}{\sin 78^\circ} = \frac{88000}{\sin 24}, \quad \therefore \text{Compression} = 21{,}2000 \text{ kN} \rightarrow (1)$$

$$I = \frac{\pi \left[d^4 - (d-2t)^4\right]}{64} \rightarrow (2)$$

$$I = \frac{\pi \left(3660^4 - 3609.2^4\right)}{64} = 4.79 \times 10^{11} \text{ mm}^4$$

$$l_e = \sqrt{\frac{\pi^2 E I}{P_E}} = \sqrt{\frac{\pi^2 \times 200 \times 10^3 \times 4.79 \times 10^{11}}{212 \times 10^6}} = 66.8 \text{ m} \rightarrow (3)$$

Therefore, theoretically, we need to have bracing at 66.8 m distance and it will work out to be about 4 only. In reality, the section being more slender near the end we will need more. **Six bracings have been provided**. These calculation have been done by **Prof. A.D Magee, University of Bath**, in the paper **"A CRITICAL ANALYSIS OF FORTH BRIDGE "** Proceedings of Bridge Engineering 2 conference 2007, Bath ,UK. Students are advised to go through this technical paper.

The above publication deals with overview of the Forth Bridge, and the various elements of the bridge which considers the:

1) Foundations
2) Cantilevers and towers
3) Suspended spans
4) Approach Viaducts
5) Constructions
6) Sinking the foundations.

The **structural analysis** consists of involving :

1) Dead Load
2) Live Load
3) Overturning and buckling effects

Reference:------ Prof. A.D Magee, University of Bath, in the paper "A CRITICAL ANALYSIS OF FORTH BRIDGE " Proceedings of Bridge Engineering 2 conference 2007, Bath ,UK .